

## The Gram-Schmidt algorithm

1. You have found that two eigenvectors of a linear operator are  $\mathbf{v}_1 = \begin{pmatrix} 2.3 \\ 0 \\ 1 \end{pmatrix}$  and  $\mathbf{v}_2 = \begin{pmatrix} -5.3 \\ 1 \\ 0 \end{pmatrix}$ . You need to

create an orthonormal pair of vectors from this pair using Gram-Schmidt.

2. You have two normalized vectors, and each represents a signal that can be generated by a particular oscillator. It is, however, easier to find a linear combination of these two generated signals that approximates a given signal if these two signals are orthonormal. Apply Gram-Schmidt so that you know how much to adjust the second signal.

$$\hat{\mathbf{v}}_1 = \begin{pmatrix} 0.5 \\ 0.3535 \\ 0 \\ -0.3535 \\ -0.5 \\ -0.3535 \\ 0 \\ 0.3535 \end{pmatrix} \text{ and } \hat{\mathbf{v}}_2 = \begin{pmatrix} 0.5 \\ 0.3535 \\ 0 \\ -0.3535 \\ -0.5 \\ -0.3535 \\ 0 \\ 0.3535 \end{pmatrix}.$$

3. Three cameras are pointing in different directions, but it would be ideal if they were pointing in directions perpendicular to each other. The first camera should stay fixed, the second should rotate relative to the first, and the third should move relative. After you determine the new directions the vectors should be pointed, how many degrees must the second camera be moved away from the first, and then how many degrees must the third camera be moved away from both the first and second?

$$\hat{\mathbf{u}}_1 = \begin{pmatrix} 0.80 \\ 0.36 \\ -0.48 \end{pmatrix}, \hat{\mathbf{u}}_2 = \begin{pmatrix} 0.64 \\ -0.48 \\ 0.6 \end{pmatrix} \text{ and } \hat{\mathbf{u}}_3 = \begin{pmatrix} 0 \\ 0.6 \\ 0.8 \end{pmatrix}.$$

Print the entries of the resulting vectors to six significant digits.